Waves on the surface of a vapor film under conditions of intensive heat fluxes

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We investigate surface waves on the interface between a thin vapor film and a layer of liquid in the presence of a high steady heat flux. This problem arises when a metal surface heated to a high temperature is immersed into a cold liquid. The general boundary conditions, which take into account the temperature dependence of saturation pressure on the vapor-liquid interface, are derived. These boundary conditions generalize the traditional conditions on the free surface of liquid in the gravity field. The stability of the planar vapor-liquid interface is investigated analytically with linear approximations. The dispersion equation for surface waves on the vapor-liquid interface in the presence of strong heat flux is derived. A number of different, distinct from the classical surface wave problem, effects arise in the problem under consideration. The thermal processes, which occur on the phase boundary and are possible in the absence of gravity force, lead to the generation of weakly decaying periodic waves of low amplitude, whose velocities may exceed significantly those of gravity waves. The heat flux through the interface may cause periodic surface waves of small length (ripple), which are not capillary. The processes of phase transition on the interface are capable of providing the stability of vapor film under a layer of liquid in the gravity field.

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I. INTRODUCTION

Effects that differ significantly from those occurring on a free surface of liquid in the gravity field may occur on the surface of vapor film emerging between a hot material and a cooling liquid even under conditions of stable stratification of the media. Examples of processes occurring when solid or liquid medium heated to a high temperature comes in contact with cold liquid include heavy accidents at nuclear power plants and chemical factories or submarine volcanic eruptions. Intensive investigations of such processes are presently under way in many countries (see [1–9] and the list of references in these works).

Oscillations and the explosive mechanism of the breakdown of vapor film was observed by Glazkov *et al.* in experiments [1–3] as a heated solid metal hemisphere immersed into a layer of cold liquid (pressure of 1 atm and water temperature of 293 K). Video recording revealed surface oscillations of the vapor film (about 200 μm thick) on the hemisphere surface. Intervals between the frames 1/25 and 1/50 s (the exposure time in video filming did not exceed $\sim 10^{-4}$ s) enabled them to observe relatively fast processes invisible to the naked eye. The first frames ($\sim$30 s from the instant of immersion of the hemisphere into water) show the beginning of vapor film formation. As time went on the waves appeared on the vapor-liquid interface. In several cases these waves were proceeded by low frequency (2 Hz) fluctuations of the smooth thickness of vapor film. A stable film with small amplitude waves was observed on two types of the heater surface: on the surface, newly cleaned from oxides, and the other with a thick oxide layer (more then 100 μm) of low heat conductance. The last type of heater surface may be obtained after the process of coating within several minutes. In experiments with oxide layers, bubbles originated on local parts of the heater surface and within 0.1 to 0.2 s bubble boiling was spreading over the entire surface of the hemisphere. Subsequent frames show the process of spherical expansion of the vapor region and the transition to nucleate boiling.

In this paper the surface waves of small amplitude on the interface between a thin vapor film and a layer of liquid are investigated in the presence of a steady heat flux from a metal surface heated to a high temperature to the vapor film and then from vapor to subcooled liquid. The stability of the planar vapor-liquid interface is studied using linear approximations. We derive generalized boundary conditions, which take into account the temperature dependence of saturation pressure on the vapor-liquid interface. The dispersion equation for surface waves on the liquid-vapor interface in the presence of strong heat flux is derived. The thermal processes, which occur on the phase boundary and are possible in the absence of the force of gravity as well, lead to the generation of weakly decaying periodic waves of low amplitude, whose velocity may exceed significantly that of gravity waves. The heat flux through the interface may cause on this surface periodic waves of small length (ripple) which are not capillary. The wave processes on the phase interface are capable of providing for the stability of thin vapor film under the layer of liquid in the gravity field. Capillary-gravity surface waves may arise in the field of inertial forces (gravity) on the liquid-vapor interface, similar to any gas-liquid interface. As distinct from effects on a free surface of isothermal liquid, new additional effects may arise in the processes of boiling where intense heat fluxes exist which proceed from a metal surface heated to high temperature to a vapor film and then from vapor to cold liquid. In the case of high temperature gradients (in this case, over 10^7 deg/m) and accordingly high heat fluxes, the processes of evaporation and condensation on the interface both affect significantly the dynamics of capillary-gravity surface waves and lead to the generation of disturbances of a different type. By understanding the nature of the small amplitude waves on the interface between the vapor film and the liquid we can explain the behaviors of finite amplitude waves and explosive vanishing of the vapor film that were seen in experiments [1–5]. From these experi-

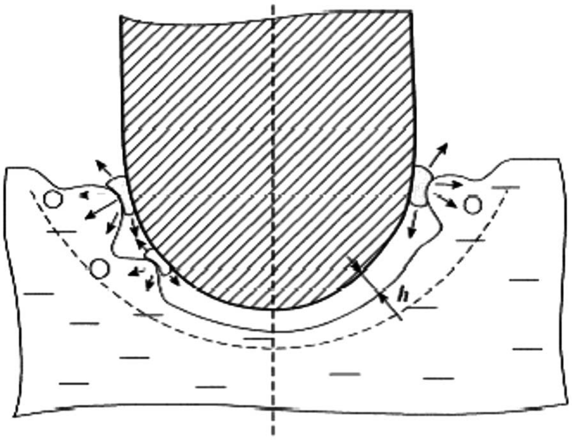

FIG. 1. Two-phase system consisting of a steady-state vapor film of finite thickness h located above a thick layer of cold liquid when a heated sphere is partly immersed in liquid.

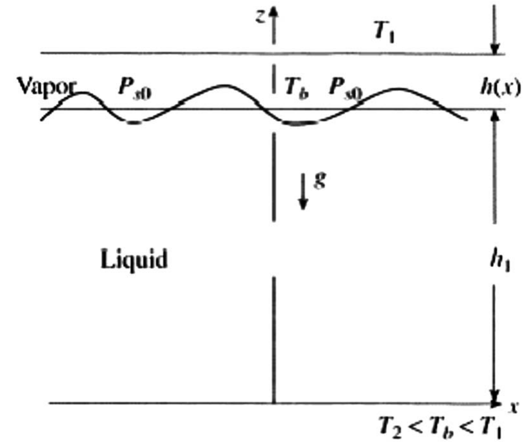

FIG. 2. Two-phase system consisting of a steady-state vapor film of finite thickness and a planar heater located above the layer of liquid.

ments and our analysis of waves of small amplitude, we could conclude that periodic waves of finite amplitude, specific solitons, and the instabilities of finite amplitude waves may arise in the nonlinear stage of the vapor film evolution. Preliminary results of the explosive instability when the amplitude of the initial small surface wave grows indefinitely at a finite time were presented in our previous work [6].

II. SURFACE WAVES ON THE LIQUID-VAPOR INTERFACE

A. Steady state of the vapor film

As mentioned above, in the experiments [1–3] a metal sphere heated to a temperature higher than the boiling temperature of a liquid was placed in a container filled with that liquid at rest. When the liquid was heated and set to boiling, a film of vapor formed at the sphere surface of the heater. With the problem of temperature distribution in vapor and liquid simplified, the coordinate and velocity of motion of the vapor-liquid interface should be determined from the solution of the Stefan problem. If the heated sphere is immersed in liquid only partially (Fig. 1), as was the case in the experiments [1–3], a steady-state boiling with a weakly varying film thickness is possible. Here we consider regimes, where only a minor part of vapor goes out to the space above the heater, and the bulk of heat released by the heating surface is transferred from vapor to liquid and then from liquid to the environment.

We will perform a simplified analysis of the processes observed in the experiments [1–3] in Cartesian coordinates ($-h_1 \leqslant z \leqslant h$, $-\infty < x < \infty$) while investigating the stability of a planar two-phase system consisting of a stationary vapor film of finite thickness h (Fig. 2) and a thick layer ($h_1 \gg h$) of cold liquid located beneath this film. We will consider the case where the temperature of liquid, except for its surface, is lower than the boiling temperature, the top surface of the container is maintained at constant temperature T_1 exceeding the boiling temperature of liquid $T_b(P)$ at pressure P, and the bottom wall of the container is at temperature T_2 which is lower than the boiling temperature of liquid: $T_2 < T_b(P) < T_1$.

If the gravity force is directed in opposition to axis z (see Fig. 2), this situation corresponds to a stable stratification of light and heavy media. However, the problem under consideration differs from the standard problem on stability of isothermal layers of light and heavy media in the gravity field. This difference consists both in the fact that the system is not isothermal and in the fact that mass fluxes may arise on the interface because of the boiling of liquid or the condensation of vapor. The mode of operation with steady-state thickness of vapor film is possible only in the case of well-defined values of heat flux from heated surface $q_w = q_0$. In this mode of operation the transient process of liquid boiling and of vapor generation results in the formation of a steady-state two-phase system with the vapor film of finite thickness disposed on the layer of a liquid. No evaporation of liquid occurs in such steady state, and the vapor pressure P_s is equal to saturation pressure $P_b(T_b)$ corresponding to the boiling temperature on the surface of liquid: $T_L = T_b$.

Steady-state distributions of temperature in vapor and in liquid at rest (its level corresponds to coordinate $z=0$, and the bottom is at $z=-h_1$) can be readily obtained from the solution of one-dimensional heat equations

$$\frac{d}{dz}\lambda_s \frac{d}{dz} T_{s0} = 0 \quad \text{and} \quad \frac{d}{dz}\lambda_L \frac{d}{dz} T_{L0} = 0. \tag{1}$$

These equations are solved for the following boundary conditions on the container walls:

$$z = h, \quad T_s = T_1 \gg T_2; \tag{2}$$

$$z = -h_1, \quad T_L = T_2 \ll T_1; \tag{3}$$

and on the $z=0$ interface

$$T_s(0) = T_L(0) = T_b(P_b), \tag{4}$$

$$\lambda_L \frac{d}{dz} T_L = \lambda_s \frac{d}{dz} T_s = -q_0 = \text{const} > 0. \tag{5}$$

Here T_b is the boiling temperature, P_b is the boiling pressure, and λ_L, λ_s are thermal conductivities of liquid and vapor, respectively.

If the heat flux from vapor on the vapor-liquid interface is equal to the heat flux to liquid then the mass flux due to evaporation of liquid or condensation of steam is absent. Linear profiles of temperature in liquid and vapor are obtained for constant values of thermal conductivity coefficients. The solution of problems (1)–(5) may be readily obtained in view of thermal conductivity coefficients dependence on the temperature as well using the Kirchhoff variable. However, the temperature dependence of thermal conductivity coefficients is of no importance in this treatment.

B. Equations and boundary conditions for nonstationary disturbances

We will now investigate the dynamics of the two-dimensional disturbances of the steady state considered above, where $h \ll h_1$. The subscripts L and s indicate various properties of liquid and vapor, respectively: thermal diffusivity, thermal conductivity, density, heat capacity, pressure, and velocity potential. The subscript 0 indicates various properties of liquid and vapor in stationary condition. The subscript b indicates various boiling properties, such as the boiling temperature T_b and the boiling pressure P_b. The subscript F indicates the velocity of the vapor-liquid interface in the direction of its normal. The subscript f indicates the effective acceleration. The subscript cr indicates the critical length of gravity-capillary waves. The subscript n indicates the derivative ∇_n with respect to the normal to the perturbed liquid-vapor interface. t_c is the characteristic time of the problem.

In systems in which a light hot medium, i.e., vapor, is in the gravity field above liquid, waves may propagate on the interface and disturb its shape. In our case, unlike the problem of waves on a free isothermal surface, one must take into account the temperature nonuniformity in vapor and liquid and the processes of evaporation and condensation on the interface between liquid and vapor. We will first investigate the evolution of the small disturbances using the equations of continuity, motion, and energy for each one of the media. This problem is closely connected with the problem of stability of the steady state relative to small disturbances.

The disturbed shape of the liquid-vapor interface $z=\xi(x,t)$ must be determined during the process of solving the problem. The liquid may be assumed to be incompressible, and its motion at velocity $\mathbf{u}=(u,0,w)$ may be assumed to be irrotational. Of course, vapor is not an incompressible medium; however, in the case where the ratio of the characteristic velocity of vapor u_s to the velocity of sound in vapor c_s is low,

$$u_s/c_s \ll 1, \tag{6}$$

unsteady-state effects may be ignored in the equations of continuity and motion. We ignore the effect of viscous forces in both media. We introduce velocity potentials for liquid and vapor (functions ψ and ψ_s, respectively)

$$\mathbf{u}_L = \nabla\psi, \quad \mathbf{u}_s = \nabla\psi_s. \tag{7}$$

The Laplace equations obtained from the equations of continuity after substitution of Eq. (7) into the latter are used to find the functions $\psi(x,z,t)$ and $\psi_s(x,z,t)$

$$\Delta\psi = 0, \tag{8a}$$

$$\Delta\psi_s = 0, \tag{8b}$$

where Δ is the Laplacian.

The distribution of temperature disturbances in liquid T_L' is found from the heat transfer equation

$$\frac{\partial}{\partial t} T_L' - \chi_L \Delta T_L'(x,z,t) = -(\mathbf{u}\cdot\nabla) T_L'(x,z,t) = -\frac{\partial}{\partial z}\Psi \frac{\partial}{\partial z} T_{0L}. \tag{9}$$

The distribution of temperature in vapor T_s' is determined similarly using

$$\frac{\partial}{\partial t} T_s'(x,z,t) - \chi_s \Delta T_s'(x,z,t) = -\frac{\partial}{\partial z}\Psi_s \frac{\partial}{\partial z} T_{0s}, \tag{10}$$

where $\chi_j = \lambda_j/\rho_j C_{Pj}$ $(j=L,s)$ is thermal diffusivity, λ_j is thermal conductivity, ρ_j is density, and C_{Pj} is heat capacity $(j=L,\ s)$.

It is assumed that the wall temperature is maintained constant; therefore, Eqs. (9) and (10) are solved with the following boundary conditions on the channel walls:

$$T_L'(x,t,z) = -h_1 = 0, \tag{11a}$$

$$T_s'(x,t,z=h) = 0, \tag{11b}$$

and the following conditions on the disturbed interface of $z=\xi(x,t)$:

$$T_L'(x,t,z=\xi) = T_s'(\xi) = (\partial T/\partial P)_{b0}\delta P_b. \tag{12}$$

The mass flux can be calculated from [10,11]

$$\Lambda \dot{m} = -\lambda_s \nabla_n T_s' + \lambda_L \nabla_n T_L'. \tag{13}$$

Here, Λ is the specific heat of evaporation (condensation), and ∇_n is the derivative with respect to the normal to the perturbed liquid-vapor interface. The term $(\partial T/\partial P)_{b0}\delta P_b$ on the right-hand side of Eq. (12) allows for the variation of boiling temperature during the variation of vapor pressure. In order to calculate the mass flux, one must find the distributions of temperature in vapor and liquid. In the steady state, the mass flux $\dot{m}_0$ according to Eq. (5) is zero,

$$\dot{m}_0 = \left(\lambda_s \frac{d}{dz} T_{0s} - \lambda_L \frac{d}{dz} T_{0L}\right) \Big/ \Lambda = 0. \tag{14}$$

However, the disturbance of the interface may result in the emergence of mass flux, for which the following preliminary estimate may be obtained with the constraint $\xi > 0$, $\xi/h \ll 1$:

$$\dot{m} = \lambda_s\{[T_1 - T_b]/(h-\xi) - [T_1 - T_b]/h\}/\Lambda = |q_0|\xi/h\Lambda. \tag{15}$$

Its rigorous proof will be given during the construction of the complete solution of the problem. Equation (8) for the liquid velocity potential is solved with the boundary condition on the bottom of the layer of liquid,

$$z = -h_1, \quad w = \frac{\partial}{\partial z}\Psi = 0, \tag{16}$$

and with boundary conditions on the vapor-liquid interface, which will be derived later.

One of the boundary conditions on the phase interface follows from the fact that the values of vapor and liquid pressures are different because of the surface tension forces arising upon disturbance of the surface of liquid $\xi(x,t)$,

$$P_s - P_L = \sigma\frac{\partial^2}{\partial x^2}\xi. \tag{17}$$

Here, σ is the surface tension coefficient of liquid. In the general case, the surface tension coefficient depends on temperature; however, we can ignore this dependence at atmospheric pressure and assume that σ=const.

The Lagrange-Cauchy relation is valid for the potential flow of liquid,

$$\rho_L\left\{\frac{\partial}{\partial t}\psi + (1/2)\left[\left(\frac{\partial}{\partial x}\Psi\right)^2 + \left(\frac{\partial}{\partial z}\Psi\right)^2\right] + gz\right\} = -P_L, \tag{18}$$

where $P_L(t,x,z)$ is the pressure in each point of liquid. Equality (18) may be employed on the free surface of liquid as well. The vapor pressure on the interface is equal to the temperature-dependent saturation pressure $P_s = P_b(T_b)$. According to Eq. (15), the shift of the vapor film surface ξ results in disturbance of the heat flux on the interface. In turn, the variation of the heat flux from the heated surface to liquid leads to the variation of saturated vapor temperature and pressure. Therefore the variation of saturated pressure δP_s may be expanded using Taylor series in the saturated vapor temperature variation and keeping only the linear and quadratic terms in δT,

$$\delta P_s = P_s - P_{s0} = (\partial P/\partial T)_{b0}\delta T + \left(\frac{\partial^2}{\partial T^2}P\right)_{b0}(\delta T)^2 + \mathrm{O}[(\delta T)^3]. \tag{19}$$

We have $(\partial P/\partial T)_{b0} \approx 10^4$ Pa/K for water on the boiling line at atmospheric pressure; therefore we obtain the following estimate:

$$(\delta P_s/P_{s0})/(\delta T_b/T_b) = (\partial P/\partial T)_{b0}(T_b/P_{s0}) \gg 1,$$
$$(\delta T_b/T_b)/(\delta P_s/P_{s0}) \ll 1. \tag{20}$$

We use Eqs. (17) and (19) and the Lagrange-Cauchy equation to obtain the boundary condition on the liquid-vapor interface,

$$\frac{\partial}{\partial t}\psi + (1/2)\left[\left(\frac{\partial}{\partial x}\psi\right)^2 + \left(\frac{\partial}{\partial z}\Psi\right)^2\right] + g\xi - (\sigma/\rho_L)\frac{\partial^2}{\partial x^2}\xi + \delta P_s(T_s + T_s')/\rho_L$$
$$= \frac{\partial}{\partial t}\Psi + (1/2)\left[\left(\frac{\partial}{\partial x}\Psi\right)^2 + \left(\frac{\partial}{\partial z}\Psi\right)^2\right] + g\xi - (\sigma/\rho_L)\frac{\partial^2}{\partial x^2}\xi + (\partial P/\partial T)_{b0}T_s' + \left(\frac{\partial^2}{\partial T^2}P\right)_{b0}(T_s')^2/\rho_L = 0. \tag{21}$$

Here, the term $\delta P_s(T_s+T_s') = P_s(T_s+T_s') - P_s(T_s) = (\partial P/\partial T)_{b0}T_s'$ takes into account the variation of saturation pressure during the variation of the vapor temperature on the vapor-liquid interface, where g is the gravity acceleration. Equation (21) is a generalization of the well-known relation on the free isothermal surface of the gas-liquid interface [10–12] and differs from standard relations on the free surface of liquid by the term which takes into account the variation of the saturation pressure of vapor in the case of temperature disturbance.

It is known that the other boundary condition on the free surface of liquid

$$F(x,z,t) = z - \xi(x,t) = 0 \tag{22}$$

follows from kinematic considerations. If the mass flux is absent this boundary means that the velocity of motion of the disturbed interface in the direction of its normal

$$w_F = -\left(\frac{\partial}{\partial t}\xi + u\frac{\partial}{\partial x}\xi\right)\Big/\left[\left(\frac{\partial}{\partial x}\xi\right)^2 + 1\right]^{1/2}$$
$$= -\left(\frac{\partial}{\partial t}\xi + \frac{\partial}{\partial x}\Psi\frac{\partial}{\partial x}\xi\right)\left[1 + \left(\frac{\partial}{\partial x}\xi\right)^2\right]^{-1/2} \tag{23}$$

is equal to the velocity of motion of liquid in this direction

$$w(\xi) = w_F. \tag{24}$$

If the mass flux arises on the vapor-liquid interface

$$\dot{m} = \rho_L(w_L - w_F) = \rho_s(w_s - w_F), \tag{25}$$

then the kinematic boundary condition (24) must take into account this mass flux $\dot{m}$. In the laboratory coordinate system, the kinematic boundary condition (25) for liquid in view of the mass flux becomes

$$\left(w_L - \frac{\partial}{\partial t}\xi - \frac{\partial}{\partial x}\Psi\frac{\partial}{\partial x}\xi\right)\Big/\left[\left(\frac{\partial}{\partial x}\xi\right)^2 + 1\right]^{1/2} = \dot{m}/\rho_L. \tag{26}$$

Because of the mass flux arising on the vapor-liquid interface, the kinematic boundary condition also differs from the similar one on the free isothermal surface of liquid. The disturbance of the planar vapor-liquid interface may cause the generation of a flux of matter on this surface even in the absence of evaporation from the surface in the steady state (when $\dot{m}_0=0$). A flux of matter generates either a source of vapor arising on the disturbed interface in the case of prevailing effects of evaporation of liquid from the surface or a sink arising if the effects of condensation of vapor prevail. At $(\frac{\partial}{\partial x}\xi)^2 \ll 1$, the kinematic condition (26) becomes

$$w_L = \frac{\partial}{\partial t}\xi + \dot{m}/\rho_L + \frac{\partial}{\partial x}\Psi\frac{\partial}{\partial x}\xi - \frac{\partial}{\partial t}\xi\left(\frac{\partial}{\partial x}\xi\right)^2 \Big/ 2. \quad (27)$$

It follows from Eq. (21) that two additional effects must be taken into account on the interface (distinct from the boundary conditions on the free surface of isothermal liquid): the temperature dependence of saturated vapor pressure and the emergence of mass fluxes from the surface. These effects, which result in the modification of the boundary conditions on the interface, appear even in the case of a linear problem. One can easily see in Eq. (21) that the second term caused by the variation of saturated vapor pressure $(\frac{\partial^2}{\partial T^2}P)_{b0}(T_s')^2/\rho_L$ is significant only in the nonlinear stage of development of disturbances.

The Laplace equation for the potential of vapor velocity (8) must be solved with a boundary condition on the solid surface

$$z = h, \quad w_s = \frac{\partial}{\partial z}\psi_s = 0 \quad (28)$$

and with a boundary condition at $z=\xi$,

$$w_s = \frac{\partial}{\partial z}\psi_s = \frac{\partial}{\partial t}\xi + \dot{m}/\rho_s + \frac{\partial}{\partial z}\psi_s\frac{\partial}{\partial x}\xi - \frac{\partial}{\partial t}\xi\left(\frac{\partial}{\partial x}\xi\right)^2 \Big/ 2. \quad (29)$$

C. Linear problem for small disturbances evolution

We will now consider the stability of a planar steady-state interface relative to small disturbances. We will seek the solution of a linear problem for small disturbances in the form

$$\xi(x,t) = \xi_{00}\exp i(kx+\omega t),$$
$$\psi(x,z,t) = f_0(z)\exp i(kx+\omega t),$$
$$\psi_s(x,z,t) = f_s(z)\exp i(kx+\omega t),$$
$$T_L' = f_L(z)\exp i(kx+\omega t), \quad T_s' = f_s(z)\exp i(kx+\omega t). \quad (30)$$

Here ω is the fluctuation frequency, and k is the wave number $(-\infty < k < \infty)$.

The problem on disturbed motions in liquid and vapor may be solved first, followed by obtaining the distribution of temperature disturbances in vapor and liquid and the behavior of disturbances of the interface. The solution of the Laplace equation (8a) for the velocity potential of liquid with boundary condition (16) has the form

$$\psi(x,z,t) = [f_{00}/\sinh(kh_1)]\cosh[k(z+h_1)]\exp i(kx+\omega t), \quad (31)$$

where f_{00} is an unknown constant. We similarly find from the Laplace equation (8b) the velocity potential of vapor with boundary condition (28) on a solid surface

$$\psi_s(x,z,t) = -[f_{0s}/\sinh(kh)]\cosh[k(h-z)]\exp i(kx+\omega t), \quad (32)$$

where f_{0s} is an unknown constant.

The disturbance of the phase interface has the form

$$\xi(x,t) = \xi_{00}\exp i(kx+\omega t), \quad (33)$$

where ξ_{00} is an unknown constant.

According to Eq. (1) stationary temperature profiles in vapor and liquid are given by

$$T_{0L}(z) = T_b - (T_b - T_1)(z/h_1) = T_b + |q_0|(z/\lambda_L) \quad (34)$$

and

$$T_{0s}(z) = T_b - (T_b - T_w)(z/h) = T_b + |q_0|(z/\lambda_s). \quad (35)$$

Given the velocity fields in vapor and liquid, we can find the temperature fields from Eqs. (9) and (10). For small disturbances Eqs. (9) and (10) become

$$\frac{\partial}{\partial t}T_L' - \chi_L\Delta T_L'(x,z,t) = -k[f_{00}/\sinh(kh_1)](|q_0|/\lambda_L)\sinh[k(z+h_1)]\exp i(kx+\omega t) \quad (36)$$

and

$$\frac{\partial}{\partial t}T_s' - \chi_s\Delta T_s'(x,z,t) = -k[f_{0s}/\sinh(kh)](|q_0|/\lambda_s)\sinh[k(h-z)]\exp i(kx+\omega t). \quad (37)$$

The obtained solutions of Eqs. (36) and (37) satisfy the boundary conditions (12) on the disturbed interface $z=\xi(x,t)$. Considering inequality (20), we can ignore the variation of boiling temperature during the variation of vapor pressure and write the boundary conditions for disturbances of temperature (12) at $z=0+\xi(x,t)$ as

$$T_L'(xt,z=0) + (dT/dz)_{L0}\xi = T_L'(x,t,z=0) + (q_0/\lambda_L)\xi = 0, \quad (38)$$

$$T_s'(xt,z=0) + (dT/dz)_{s0}\xi = T_s'(x,t,z=0) + (q_0/\lambda_s)\xi = 0. \quad (39)$$

We substitute the obtained value of velocity (31) into the heat transfer equation for liquid (36) and the solution of Eq. (32) into the heat transfer equation for vapor (37), and use boundary conditions (11a), (38), (11b), and (39) to find the distribution of temperature disturbances in liquid and vapor. The general solution of Eqs. (36) and (37) contains the solutions of homogeneous and nonhomogeneous equations,

$$T_L'(xt,z=0) = -(|q_0|/\lambda_L)\{[\xi_{00} - (f_{00}k/i\omega)]\sinh^{-1}(\alpha_L h_1) \times \sinh[\alpha_L(z+h_1)] + (f_{00}k/i\omega)\sinh(kh_1)^{-1} \times \sinh[k(z+h_1)]\}\exp i(kx+\omega t) \quad (40)$$

and

$$T_s'(x,t,z) = -(|q_0|/\lambda_s)\{[(\xi_{00} - f_{0s}k/i\omega)/\sinh(\alpha_s h_1)] \times \sinh[\alpha_s(h-z)] + (k/i\omega)[f_{0s}/\sinh(kh)] \times \sinh[k(h-z)]\}\exp i(kz+\omega t), \quad (41)$$

where

$$\alpha_j = (k^2 + i\omega/\chi_i)^{1/2}, \quad j = L,s.$$

It is assumed that the following inequalities are valid for the characteristic time of the problem $t_c \approx 1/\omega$ and times of relaxation to steady-state distribution of temperatures in liquid $t_L = 1/k^2\chi_L$ and in vapor $t_s = 1/k^2\chi_s$:

$$t_s/t_c < 1, \quad t_L/t_c < 1. \quad (42)$$

Using distributions of temperature disturbances in liquid (40) and vapor (41), we find the disturbance of mass flux according to Eq. (13),

$$\Lambda\dot{m} = -\left(\lambda_s \frac{d}{dz}T_s'\right)_{z=0} + \left(\lambda_L \frac{d}{dz}T_L'\right)_{z=0} = 0.$$

Using solutions (40) and (41) and calculating mass flux, we have

$$\dot{m} = (|q_0|k/\Lambda)\{(\alpha_s/k)[(\xi_{00} - f_{0s}k/i\omega)]\coth(\alpha_s h) + (f_{0s}k/i\omega)\coth(kh) + [(\alpha_L/k)(\xi_{00} - f_{00}k/i\omega)\coth(\alpha_L h_1) + (f_0 k/i\omega)\coth(kh_1)]\}\exp i(kx+\omega t). \quad (43)$$

Using boundary conditions (25), we can express the vapor velocity $w_s(z=0)$ and the liquid velocity $w_L(z=0)$,

$$w_L = \frac{\partial}{\partial t}\xi + \dot{m}/\rho_L, \quad w_s = \frac{\partial}{\partial z}\psi_s = \frac{\partial}{\partial t}\xi + \dot{m}/\rho_s.$$

Using this equation, the constant f_{0s} in Eq. (43) can be expressed through constants f_{00} or ξ_{00}. Since $\rho_s/\rho_L \ll 1$, we obtain

$$f_{00} = i\omega\xi_{00} \quad \text{and} \quad (44a)$$

$$f_{0s}\exp i(kx+\omega t) = f_{00}\exp i(kx+\omega t) + (\dot{m}/k\rho_s). \quad (44b)$$

Substituting the relation (44b) in Eq. (43) we obtain the following expression for the mass flux, which is more suitable for the next analysis:

$$\dot{m} = -(|q_0|k/\Lambda)\{(\xi_{00} - f_{00}k/i\omega)[(\alpha_s/k)\coth(\alpha_s h) + (\alpha_L/k)\coth(\alpha_L h_1)] + (f_{00}k/i\omega)\coth(kh) + (f_0 k/i\omega)\coth(kh_1)\}\{1 + (|q_0|k/i\Lambda\rho_s\omega)[\coth(\alpha_s h) - \coth(kh)]\}^{-1}\exp i(kx+\omega t). \quad (45)$$

Now we can calculate the characteristic speed of the liquid mass flux,

$$\dot{m}/\rho_L = -(|q_0|k/\Lambda\rho_L)\{(\xi_{00} - f_{00}k/i\omega)[(\alpha_s/k)\coth(\alpha_s h) + (\alpha_L/k)\coth(\alpha_L h_1)] + (f_{00}k/i\omega)\coth(kh) + (f_0 k/i\omega)\coth(kh_1)\}\{1 + (|q_0|k/i\Lambda\rho_s\omega)[\coth(\alpha_s h) - \coth(kh)]\}^{-1}\exp i(kx+\omega t).$$

The previous equation can be rewritten as

$$\dot{m}/\rho_L = [Qc(\omega,k)\xi_{00} + i(Qk/\omega)d(\omega,k)f_{00}]\exp i(kx+\omega t), \quad (46)$$

where

$$Q = |q_0|k/\rho_L\Lambda, \quad (47)$$

$$\mathrm{c} = [(\alpha_s/k)\coth(\alpha_s h) + (\alpha_L/k)\coth(\alpha_s h)]\{1 + (|q_0|k/i\Lambda\rho_s\omega) \times [\coth(\alpha_s h) - \coth(kh)]\}^{-1}, \quad (48)$$

$$\mathrm{d} = [(\alpha_s/k)\coth(\alpha_s h)(\alpha_L/k)\coth(\alpha_L h_1) - \coth(kh) - \coth(kh_1)] \times \{1 + (|q_0|k/i\Lambda\rho_s\omega)[\coth(\alpha_s h) - \coth(kh)]\}^{-1}. \quad (49)$$

D. Dispersive equation for surface waves on the vapor-liquid interface

Using Eq. (39) we can eliminate the disturbance of vapor temperature from boundary condition (21). Then, using Eq. (46), we can reduce boundary conditions (21) and (27) to the form which depends only on the hydrodynamic potential of liquid $\psi(x,z=h,t)$ and on the shift of the interface $\xi(x,t)$.

$$\frac{\partial}{\partial t}\psi + g_f\xi - (\sigma/\rho_L)\frac{\partial^2}{\partial x^2}\xi = i\omega f_{00}\coth(kh_1) + (g_f + \sigma k^2/\rho_L)\xi_{00} = 0 \quad (50)$$

and

$$\frac{\partial}{\partial t}\psi - \frac{\partial}{\partial t}\xi - (\dot{m}/\rho_L) = k(1 - iQd/\omega)f_{00} - (i\omega + Qc)\xi_{00} = 0, \quad (51)$$

where

$$g_f = g + q_0(\partial P/\partial T)_{b0}/\rho_L\lambda_s. \quad (52)$$

Equations (50) and (51) yield the dispersion equation for disturbances of the liquid-vapor interface

$$\mathrm{D}(\omega,k) \equiv \omega^2 - i\omega Qc - k\tanh(kh_1)(g_f + \sigma k^2/\rho_L)(1 - iQd/\omega) = \omega[\omega^2 - i\omega Qc - k\tanh(kh_1)(g_f + \sigma k^2/\rho_L)] + ik\tanh(kh_1)(g_f + \sigma k^2/\rho_L)Qd = 0. \quad (53)$$

It is clear that, unlike the known problem on the free surface of an isothermal liquid in the gravity field [11,12], in the problem considered here the dispersive equation for surface waves has an additional root. The dispersive equation (53) may be rewritten in dimensionless form, using the following dimensionless variables:

$$\Omega = \omega(g_f k)^{-1/2}, \quad m = (kg_f)^{-1/2}|q_0|k/\rho_L\Lambda \ll 1, \quad \Xi = \sigma k^2/\rho_L g_f. \quad (54)$$

Now the cubic dispersive equation (53) is rewritten in the form

$$\mathrm{D}(\Omega,kh) \equiv \Omega[\Omega^2 - (1+\Xi)\tanh(kh_1)] - im[\Omega^2 c - (1+\Xi)d\tanh(kh_1)] = 0. \quad (55)$$

The dispersive equation (53) has three roots. In the case of characteristic parameters of the problem under consideration we obtain the inequality

$$m = (|q_0|k/\rho_s\Lambda\omega) \ll 1. \quad (56)$$

For water boiling under conditions of atmospheric pressure and heat flux from heated surface $q_0=10^7\ \mathrm{W/m^2}$ we have $m=3\times10^{-7}$. All roots of the dispersive equation (53) can now be obtained readily using perturbation theory:

$$\Omega = \Omega_0 + m\Omega_1 + m^2\Omega_2 + \mathrm{O}(m^3). \quad (57)$$

The zero-order solutions are

$$\Omega_{01,2} = \pm[\tanh(kh_1)(1+\Xi)]^{1/2}, \quad \Omega_{03}=0. \quad (58)$$

$$0 \leqslant \Omega_{01,2} \leqslant 1. \quad (59)$$

Roots $\Omega_{01,2}$ are similar to capillary-gravitational waves on the free surface of the isothermal liquid, but *these new waves can exist in absence of the gravity force* even when the heavy liquid is placed above light vapor if

$$g_f > 0 \quad \text{or } (\partial P/\partial T)_{b0} q_0|/\rho_L\Lambda > g. \quad (60)$$

The fluctuation frequency in a dimensional form for long waves ($kh \gg 1$) is

$$\omega = k(g_f + \sigma k^2/\rho_L). \quad (61)$$

The additive to the fluctuation frequency of surface waves $\Omega_{01,2}$ in the next following approximation on small parameter $m \ll 1$ becomes

$$\Omega_1 = im[c + \tanh(kh_1)d]/2. \quad (62)$$

As m and $c+\tanh(kh_1)d$ are real and positive [$c+\tanh(kh_1)d>0$] the additive Ω_1 is imaginary. Therefore the mass flux fluctuations lead to only a small ($\Omega_1 \approx m \ll 1$) attenuation of surface waves due to energy losses for evaporation.

The additional new frequency is imaginary and proportional to the small parameter m,

$$\Omega_3 = imd, \quad (63)$$

and $\Omega_1=0$ when $m=0$.

We will select the amplitude of surface disturbances ξ_{00} from Eq. (33) as an independent quantity and will use it to express all of the remaining constants appearing in the solution of the linear problem. For the case when $\sigma=0$ and $m=0$ from Eq. (50) we will obtain the constant of the velocity potential of liquid,

$$f_{00} = i\omega\xi_{00}/k. \quad (64)$$

Then according to Eq. (43) we will obtain the constants of the velocity potential of vapor

$$f_{0s} = f_{00} = i\omega\xi_{00}/k. \quad (65)$$

Now we will calculate functions $c(\omega,k)$, $d(\omega,k)$, and the mass flux using Eq. (46), (48), and (49):

$$c \cong \coth(kh) + \coth(kh_1) > 0, \quad (66a)$$

$$d = 0. \quad (66b)$$

$$\dot{m} = Qc(\omega,k)\xi_{00}\exp i(kx+\omega t). \quad (67)$$

At $kh \ll 1$ and $kh_1 \ll 1$ it follows from Eq. (67) that

$$\dot{m} \approx (2|q_0|/h\Lambda)\xi(x,t). \quad (68)$$

Equation (68) proves the estimate (15).

Now the dispersion equation for surface waves on the liquid-vapor interface in the presence of strong heat flux becomes

$$\mathrm{D}(\omega,k) \equiv \omega^2 - i\omega\mathrm{Q}c - k\tanh(kh_1)(g_f + \sigma k^2/\rho_L) = 0. \quad (69)$$

It follows from Eq. (69) that the mass fluxes from the interface leads to weak damping of surface oscillation. Let us emphasize that this damping is attributing to the fact that the decrease in the vapor film thickness causes an increase in the heat flux from vapor to liquid, additional liquid evaporation, and evaporation energy loss. In the case of characteristic parameters of water that is boiling at an atmospheric pressure, the ratio of damping increment to oscillation period is very low $[(|q_0|k/\Lambda\rho_L)/(kg_f)^{1/2} \approx 10^{-6}]$.

Equation (69) yields the oscillation frequency and the phase velocity of low-amplitude waves,

$$\omega = \pm(kg_f)^{1/2}, \quad V_f = \omega/k = \pm(g_f/k)^{1/2}. \quad (70)$$

The pattern of dispersion of waves, though supporting their similarity to gravity waves on the free surface of liquid, exhibits a number of significant differences. These waves are caused by disturbances of heat flux, saturation temperature, and, thereby, saturation pressure; it is because of this that the acceleration $g_f \approx (\partial P/\partial T)_{b0}(|q_0|/\lambda_s\rho_L)$ is similar to gravitational acceleration. It is of fundamental importance that waves of this type may arise because of disturbance of heat flux through the interface even in the absence of the gravity force, and on a vertical surface as well.

The mechanism of generation of waves on the interface under the effect of heat flux is associated with the fact that a random shift of the layer of liquid toward the heated surface (i) results in a reduced thickness of the vapor layer and in an increased heat flux from the heated body to liquid and (ii) causes an increase in saturation temperature and pressure. The increase in saturated vapor pressure leads to reverse motion of the shifted boundary away from the heating surface. The shift of the interface away from the heated body causes an increase in the vapor layer thickness and a decrease in the heat flux and saturation pressure. The decrease in the saturated vapor pressure leads to the shift of the vapor layer thickness in the opposite direction toward the heating surface. Therefore it is the variation of saturation pressure on the phase boundary that causes the emergence of a force tending to return it to its previous state. The heat flux variations, which cause additional evaporation of liquid or condensation of vapor, lead to only a weak damping of oscillation of the interface because a part of the energy is spent on evaporation.

The effect associated with the variation of saturation pressure on the phase boundary and leading to the emergence of oscillation may exist at any stratification of phases and may arise even in the absence of the gravity force, while gravity waves arise only in the case of stable stratification of phases when the light medium is located above the heavy one. In the case of fluctuation of the heat flux, the fluctuation of saturation pressure is reflected by the two last terms in Eq. (21); but for small fluctuations only the term $(\partial P/\partial T)_{b0}(|q_0|/\lambda_s\rho_L)$ in Eq. (21) is important. Therefore in the case of fluctuation of the heat flux interface, oscillations arise at any orientation of phases if condition (60) is valid. This effect may cause the emergence of waves propagating on the vapor film surface under conditions of boiling of the vertical wall of the container being heated.

Therefore the problem of waves on the vapor-liquid interface is analogous to the problem of propagation of gravity waves in a layer of liquid with a free boundary, and the gravity force is represented by the quantity g_f associated with the presence of heat flux through the interface and with the variation of saturation pressure on the phase boundary. The velocity of propagation of surface waves depends on the heat flux from the heating surface and through vapor to liquid and may exceed significantly the velocity of gravity waves. Possibly, all of the results obtained for gravity waves [5] are valid in this case as well. In the presence of heat flux, solitons produced by the fluctuation of saturation pressure may propagate on the interface.

In the case of unstable stratification of isothermal media in the gravity field, the Rayleigh-Taylor instability arises on the interface. Low-amplitude waves caused by intensive heat fluxes through the vapor-liquid interface may arise in the case of unstable stratification as well, when the light vapor film is under the layer of heavy liquid, if the inertia of evaporation exceeds the gravity force. The criterion of stabilization of Rayleigh-Taylor instability by the heat flux from vapor to liquid and of ensuring the existence of the vapor film under the layer of liquid has the form

$$|q_0| > q_{\mathrm{cr}} = \lambda_s\rho_L g/(\partial P/\partial T)_{b0}(|q_0|/\lambda_s\rho_L). \tag{71}$$

When a vapor film arises at atmospheric pressure under conditions of water boiling on a horizontal surface, heat fluxes from the heating surface of the order of $q_{\mathrm{cr}}\approx 2\times 10^2\ \mathrm{W/m^2}$ may lead to stabilization of the Rayleigh-Taylor instability and provide the existence of a light vapor phase under heavy phase of a liquid.

If the mass flux arises on the vapor-liquid interface ($m \neq 0$), then one can see from solutions of the considered problem [Eqs. (44b) and (68)] that the characteristic velocity of vapor could exceed that of liquid:

$$f_{0s}/f_{00} = (1 + |q_0|/\omega h\Lambda\rho_s) > 1. \tag{72}$$

The case $\mathrm{m}\gg 1$ takes place only for very high heat fluxes if the inequality

$$|q_0| \gg q_{\mathrm{cr1}} = (\rho_s\Lambda)^2 h(\partial P/\partial T)_{b0}/\rho_L\lambda_s \tag{73}$$

is satisfied. For water on the boiling line at atmospheric pressure, we obtain $q_{\mathrm{cr1}}\approx 10^{13}\ \mathrm{W/m^2}$. If $\mathrm{m}\ll 1$ the characteristic velocity of vapor order of the characteristic velocity of liquid

$$w_L \approx w_s. \tag{74}$$

So for $q_0\approx(10^5-10^7)\ \mathrm{W/m^2}\ll q_{\mathrm{cr1}}$, $\mathrm{m}\ll 1$ and the inequality (6) is satisfied.

The presence of heat flux through the interface may lead to yet another effect of propagation of short waves (ripple) on the free surface of liquid. These short waves are similar to gravity-capillary waves but are not the real capillary waves. Waves on the surface of isothermal liquid in the gravity field, which are of length

$$l < l_{\mathrm{cg}} = 2\pi(\sigma/g\rho_L)^{1/2}, \tag{75}$$

are capillary. Short waves (ripple), for which the surface tension is nevertheless of no importance, arise on the vapor-liquid interface at high heat fluxes $g_f \gg g$ in the range

$$l_c = 2\pi(\sigma/g_f\rho_L)^{1/2} < l < l_{\mathrm{cg}}. \tag{76}$$

These effects are important for the nonlinear behaviors of waves on the vapor-liquid interface at high heat fluxes.

III. CONCLUSIONS

We formulated and solved the problem of stability (due to small amplitude fluctuations) a of steady-state interface between a thin vapor film and a layer of liquid in the presence of a heat flux. We took into account the effect of the heat flux from a metal surface heated to a high temperature to the vapor film, and then from vapor to subcooled liquid, as well as the effect of the temperature dependence on saturation pressure. Boundary conditions were derived for disturbances of the steady vapor-liquid interface, which generalize the known correlations on the free surface of liquid in the gravity field. These boundary conditions allow for thermal disequilibrium of the processes, which is associated with the variation of the saturation pressure.

The stability of the planar vapor-liquid interface was investigated analytically using linear approximations. The dispersion equation for surface waves on the liquid-vapor interface at the presence of strong heat flux is derived. A number of new effects arise in the problem under consideration (as distinct from the classical problem).

(1) The thermal processes on the phase boundary lead to the propagation of weakly decaying waves of low amplitude, whose velocity may exceed significantly that of gravity waves.

(2) The heat flux through the interface may cause periodic waves of small length (ripple) which are not capillary.

(3) The thermal processes on the interface are capable of providing for the stable existence of the vapor phase under the layer of liquid in the gravity field.

Due to nonlinear effects, specific solitons and turbulence may arise on the vapor-liquid interface in the absence of gravity force. Along with periodic waves and solitons, new instabilities may arise in the nonlinear stage due to variation of the film thickness, the nonlinear waves interactions, and the surface wave turbulence generation.

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